Uncertainty calculation example

Let us suppose we measured intensity (I) versus distance (R), and we have two datasets:

Dataset 1

R [m]	I [W]
0.5	0.1776
0.8	0.0716
1.2	0.0375
1.7	0.0155
2.3	0.0107

Dataset 2

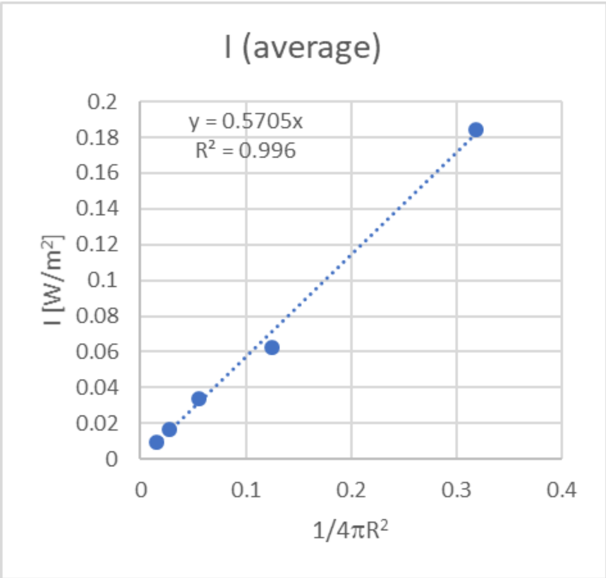
R [m]	I [W]
0.5	0.1910
0.8	0.0537
1.2	0.0305
1.7	0.0175
2.3	0.0081

Let us calculate the average, and show it versus $1/4\pi R^2$:

Average

$1/4\pi R^2$ [1/m²]	I [W]
0.3183	0.1843
0.1243	0.0627
0.0553	0.0340
0.0275	0.0165
0.0150	0.0094

Let us plot this, and determine the power from it via fitting a linear (with intersect at zero!):



This means, that the power is P = 0.571 W (rounded to three decimals). Now let us calculate the uncertainty! First, let us show the separate datasets versus $1/4\pi R^2$:

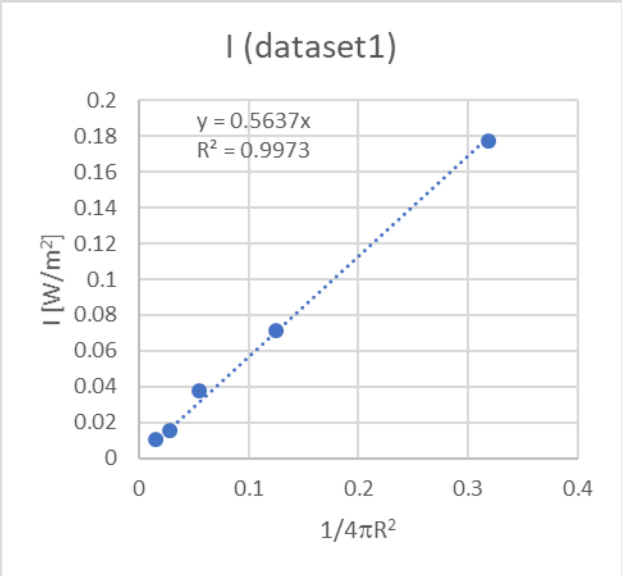
Dataset 1

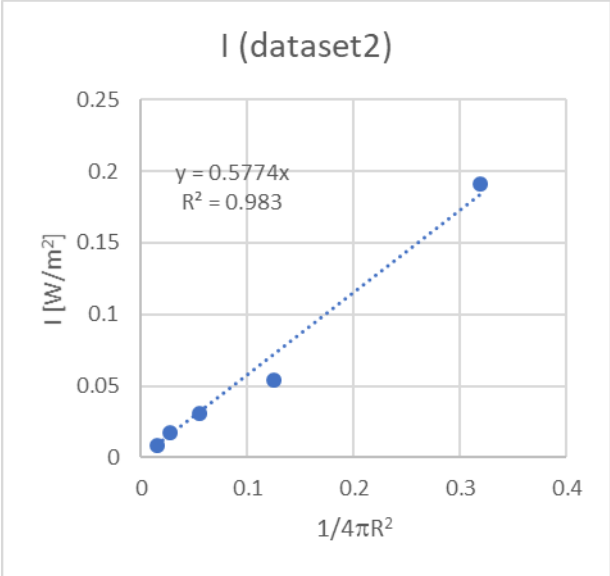
$1/4\pi R^2$ [1/m²]	I [W]
0.3183	0.1776
0.1243	0.0716
0.0553	0.0375
0.0275	0.0155
0.0150	0.0107

Dataset 2

$1/4\pi R^2$ [1/m²]	I [W]
0.3183	0.1910
0.1243	0.0537
0.0553	0.0305
0.0275	0.0175
0.0150	0.0081

Let us plot these and fit them with a linear (with intersect at zero), and determine the power:





The powers in the two cases are: P_1 = 0.564 W and P_2 = 0.577 W

This means that the uncertainty is the variance of these two numbers: ΔP = 0.007 W. This can be calculated as follows. First, we calculate the average:

$$\langle P \rangle = \frac{P_1 + P_2}{2}$$

and then the standard deviation is:

$$\Delta P = \sqrt{\frac{(P_1 - \langle P \rangle)^2 + (P_2 - \langle P \rangle)^2}{2}}$$

and similarly, if you have more than two datasets.

Hence the full result with the uncertainty is (using the P from the previous, "average" calculation, and ΔP from the uncertainty calculation):

$$P = 0.571 \pm 0.007\ W$$

Note, that you should not write out an unnecessary large number of digits, e.g. 0.570534477 ± 0.0068681 would be wrong. You should just use as many digits as the uncertainty. So, in the end, 0.57 ± 0.01 is also okay, or 0.571 ± 0.007, as I wrote above.

Also, you should observe, that while the two separate datasets deviate from the linear substantially, the average shows a nicer agreement.