Egzakt differenciálegyenletek. Integráló tényezők

A

$$p(x,y)dx + q(x,y)dy$$

differenciálegyenlet $\mu(x,y)$ integráló tényezője a

$$q(x,y)\frac{\partial \mu}{\partial x} - p(x,y)\frac{\partial \mu}{\partial y} = \mu(x,y)\left(\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}\right)$$

parciális differenciálegyenlet megoldása.

Speciális alakú integráló tényezők

Integráló tényező	Feltétel	Megoldás
$\mu = \mu(x)$	$\frac{1}{q(x,y)}\left(\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}\right) = f(x)$	$\mu(x) = \exp\left(\int f(x)dx\right)$
$\mu = \mu(y)$	$\frac{1}{p(x,y)}\left(\frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}\right) = f(y)$	$\mu(y) = \exp\left(\int f(y)dx\right)$
$\mu = \mu(x+y)$	$\frac{1}{p(x,y)-q(x,y)}\left(\frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}\right) = f(x+y)$	$\mu(x+y)$ $= \exp\left(\int f(x+y)d(x+y)\right)$
$\mu = \mu(xy)$	$\frac{1}{xp(x,y)-yq(x,y)}\left(\frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}\right) = f(xy)$	$\mu(xy) = \exp\left(\int f(xy)d(xy)\right)$
$\mu = \mu(y/x)$	$\frac{x^2}{xp(x,y)+yq(x,y)}\left(\frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}\right) = f(y/x)$	$\mu(y/x) = \exp\left(\int f(y/x)d(y/x)\right)$
$\mu = \mu(x^2+y^2)$	$\frac{1}{2(yp(x,y)-xq(x,y))}\left(\frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}\right)$ $= f(x^2+y^2)$	$\mu(x^2+y^2)$ $= \exp\left(\int f(x^2+y^2)d(x^2+y^2)\right)$

További speciális esetek:

- Ha a differenciálegyenlet homogén fokszámú és $xp(x,y) + yq(x,y) \neq 0$, akkor
$$\mu(x,y) = \frac{1}{xp(x,y) + yq(x,y)} \ .$$
- Ha
$$\frac{\partial p}{\partial x} = \frac{\partial q}{\partial y} \ , \ \frac{\partial p}{\partial y} = -\frac{\partial q}{\partial x} \ ,$$
akkor
$$\mu(x,y) = \frac{1}{p^2 + q^2} \ .$$
- Ha
$$\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x} = p(x,y)f_1(y) - q(x,y)f_2(x) \ ,$$
akkor $\mu(x,y) = m(x)n(y)$, ahol
$$m(x) = \exp\left(-\int f_2(x)dx\right) \ , \ n(y) = \exp\left(-\int f_1(y)dy\right) \ .$$