Listening to the Ear

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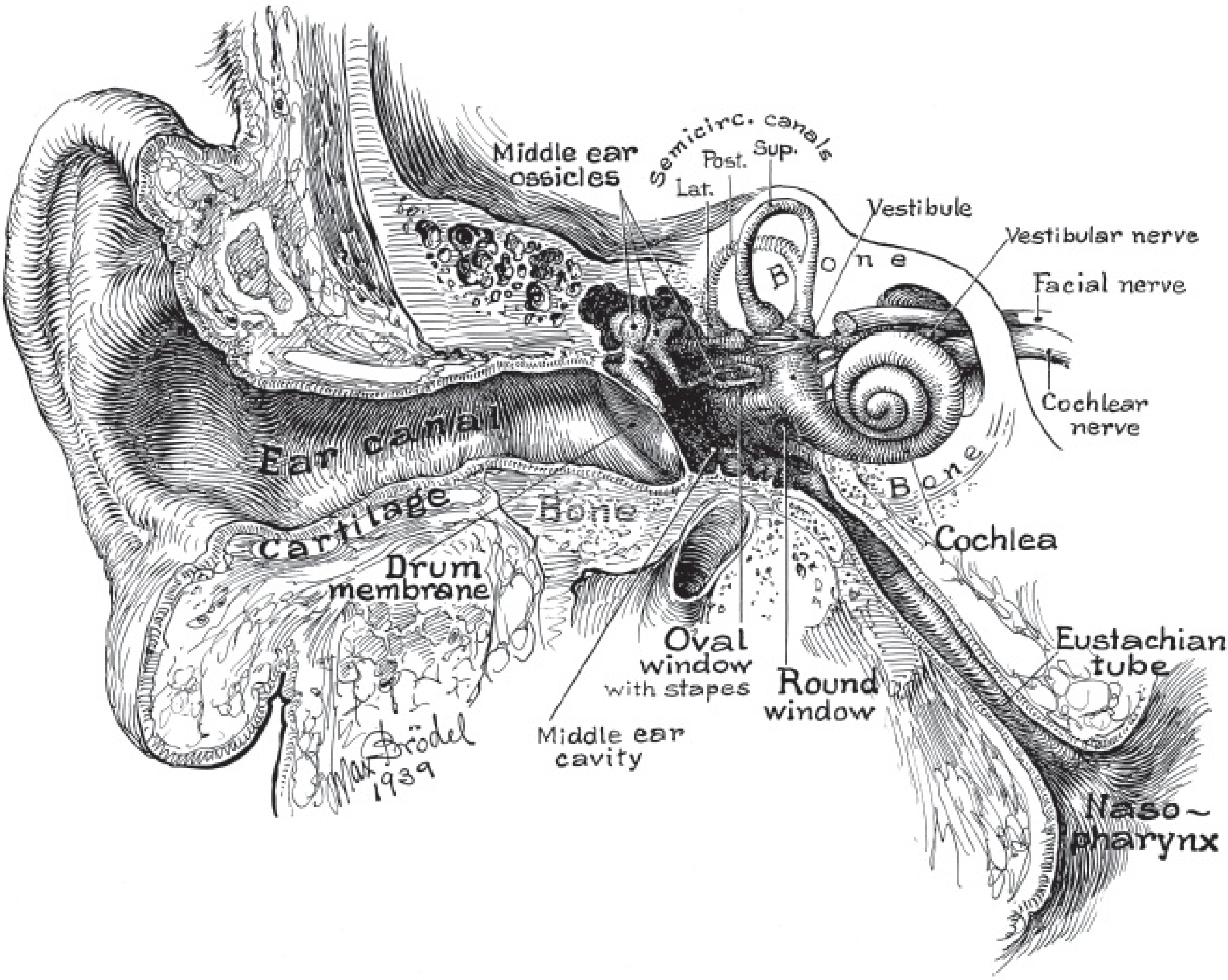
Why the ear?

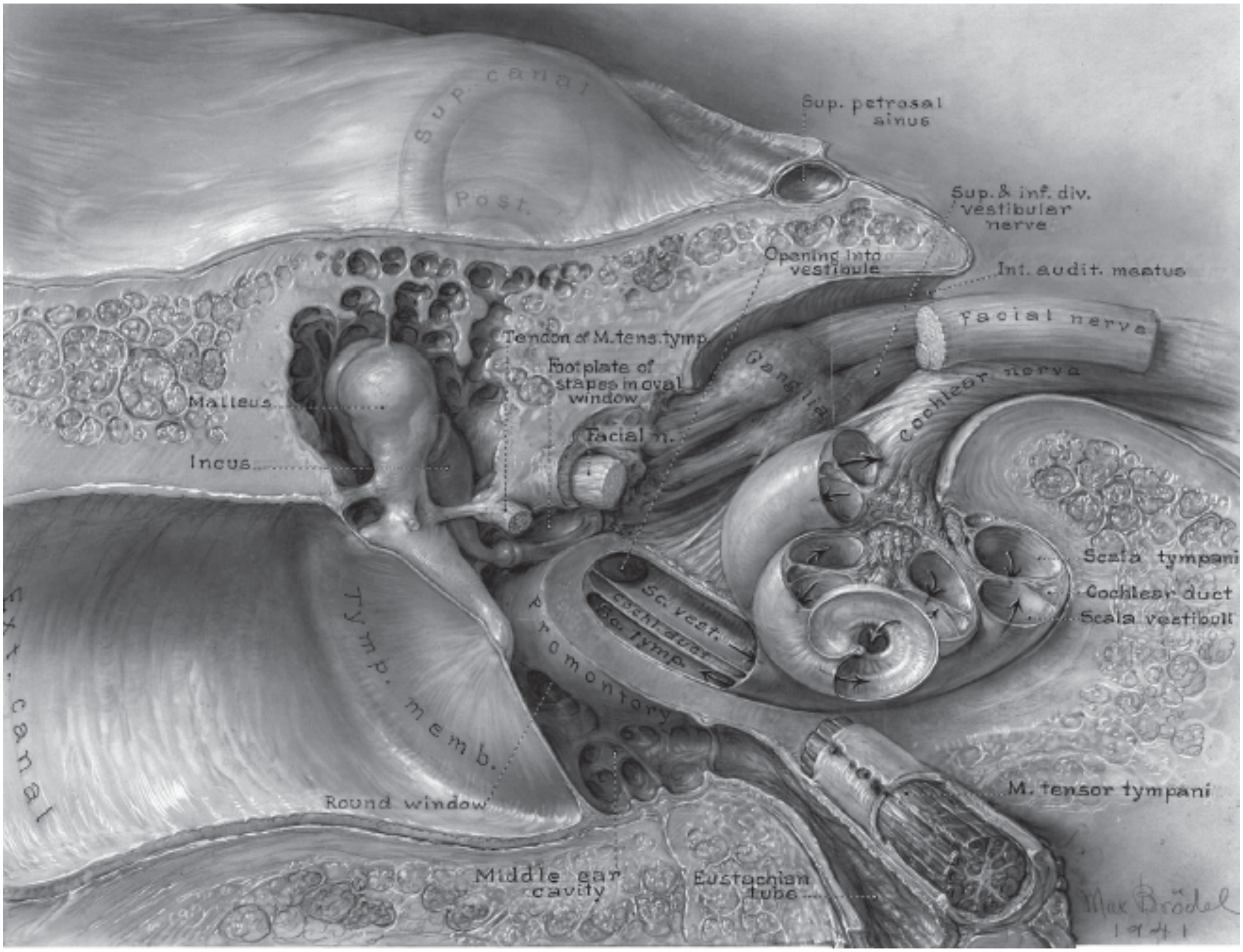
A little story:

- Strange things are happening:
  - Ears ring
  - Combination tones exist
  - Very low-level sounds are audible
    < 1Å
  - Amplitude dynamic range is large
    120 db
  - Frequency bandwidths are broad
    150 kHz

The whole is more than the sum of its parts



The external, middle, and inner ear (M. Brödel).

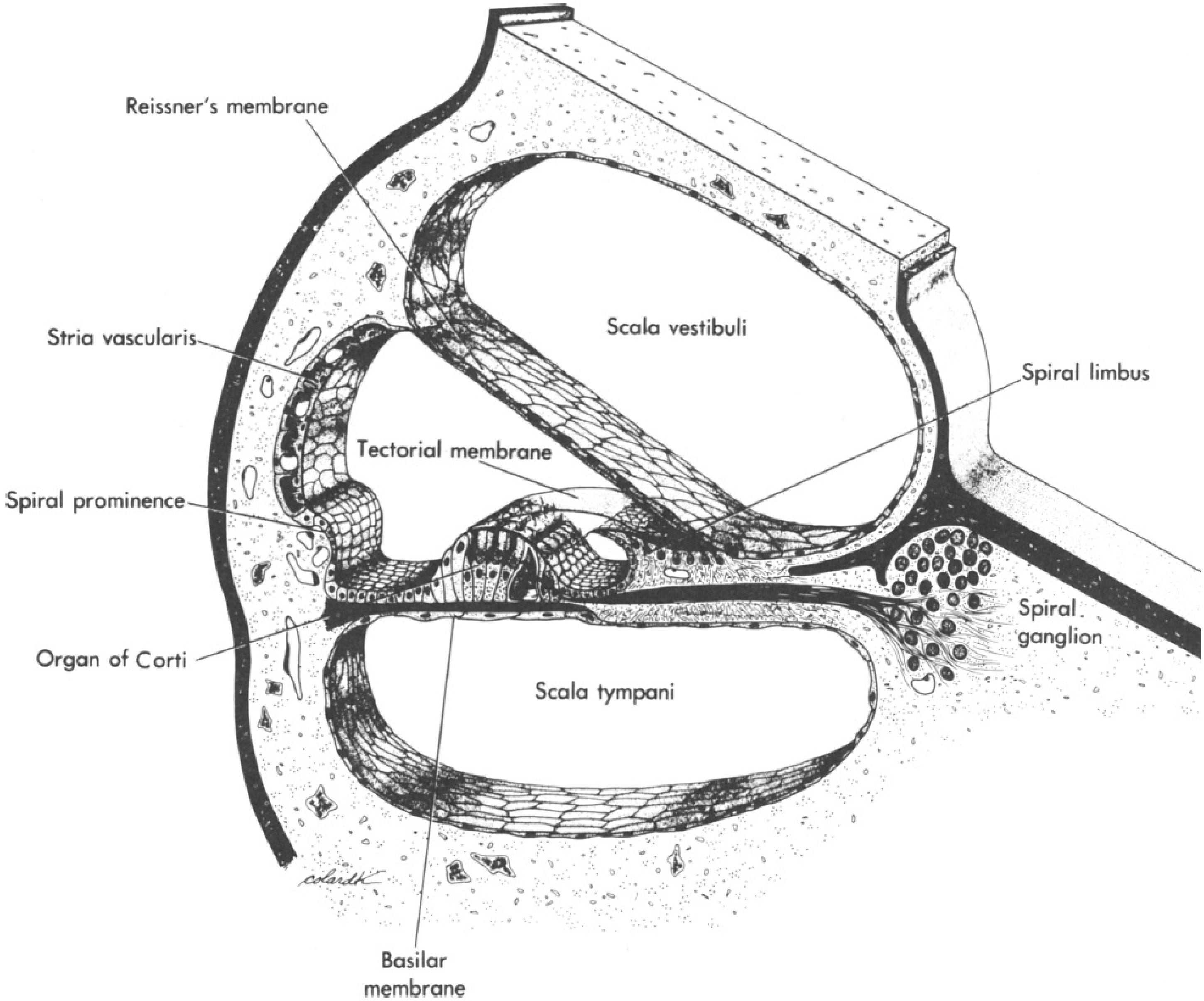


The cochlea cut open (M. Brödel).

Malleus (m): measure V_{m}

Basilar membrane (bm): measure $V_{\mathrm{bm}}(f, x, V_{\mathrm{m}})$

$$D(f, x, V_{\mathrm{m}}) \equiv V_{\mathrm{bm}}(f, x, V_{\mathrm{m}})/V_{\mathrm{m}},$$
$$T(f, x) \equiv \lim_{V_{\mathrm{m}} \to 0} D(f, x, V_{\mathrm{m}}).$$



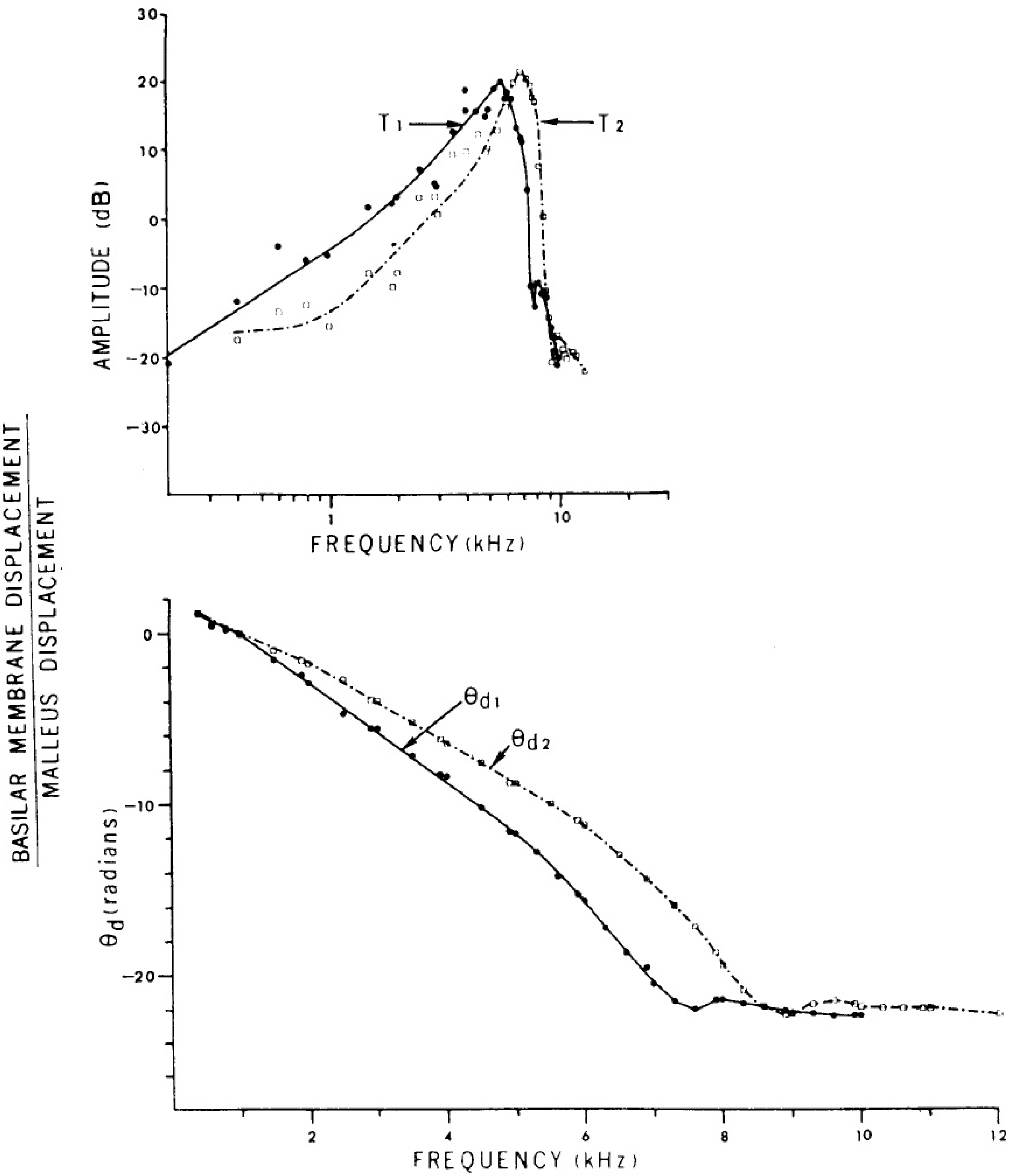
Cochlear cross section (M. Brödel).

$$Y \equiv V_{\rm bm}/P,$$
$$Y \rightarrow \text{ linear oscillator equation.}$$

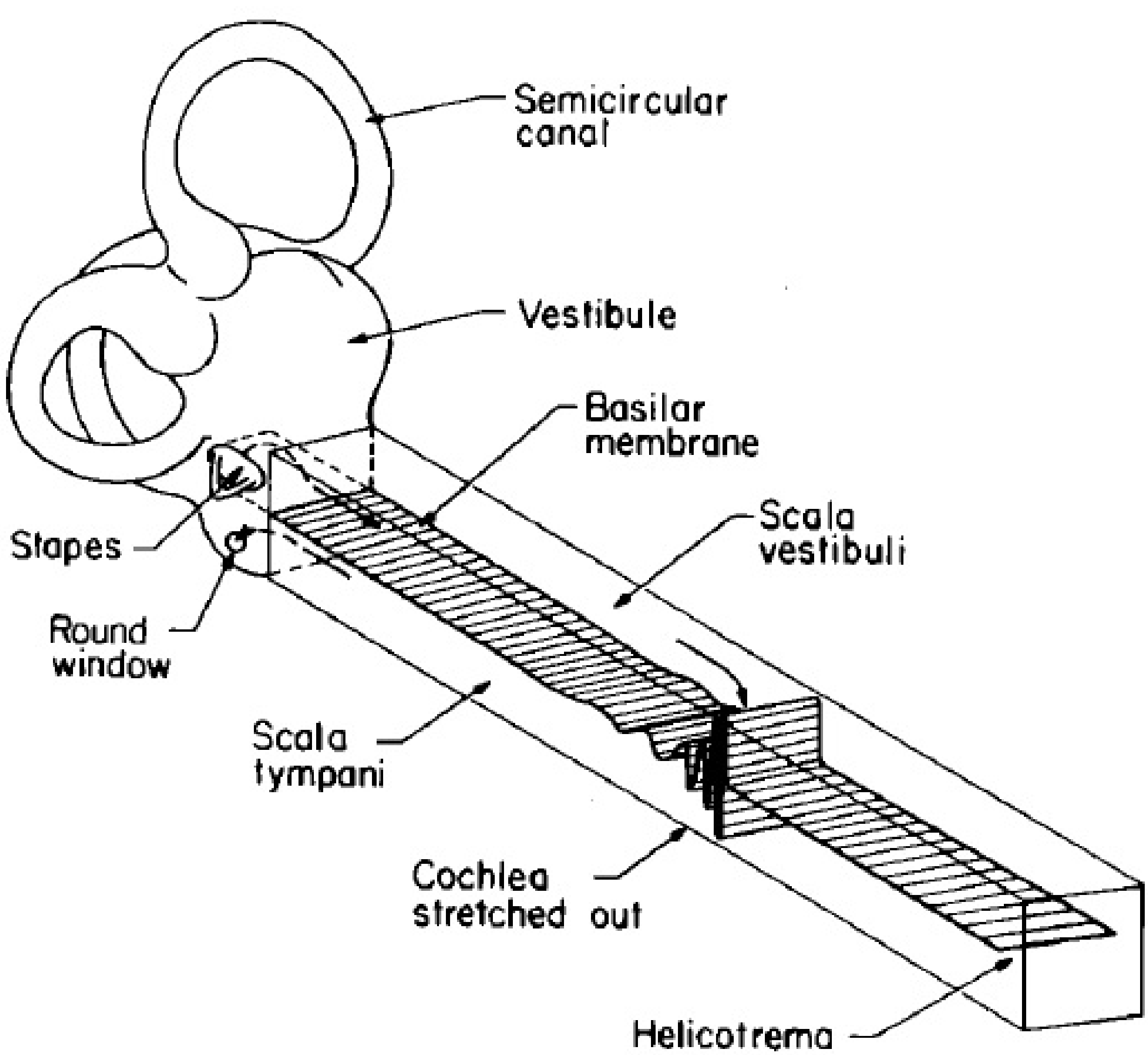
- Relatively simple gateway to the brain
  - Number of inputs is small
  - Number of outputs is small
  - Each section acts like an oscillator
  - Neurons known & don't interact
  - Neural interactions $\Rightarrow$ fluid coupling
  - Biologically significant signals are simple to describe
- Ear $\Leftrightarrow$ Mouth.
- Classical physical science is appropriate
- Abstraction is possible
- New areas in signal processing, information theory, applied mathematics, and classical physics
- A new measurement techniques available (Mössbauer effect & laser interferometry)

How does the ear work?

- Measure $D(f, x, V_{\rm m}) \equiv V_{\rm bm}(f, x, V_{\rm m})/V_{\rm m}$.



Amplitude and phase of D at two locations in the squirrel monkey (W. Rhode). $\exists\ f_c(x/l)$, $\beta \equiv f/f_c$.



The traveling wave. The cochlea is uncoiled and approximated by two fluid-filled rigid-walled compartments separated by an array of uncoupled harmonic oscillators (organ of Corti).

$N = 5$.

Two length scales: l and h,

$f_c(x/l) = f_{\max}\mathrm{e}^{-x/l}$ (from experiment).

Peak at $x \equiv \hat{x}$ or $\beta = \hat{\beta} \approx 1$ $[\beta \approx f/f_c(x)]$.

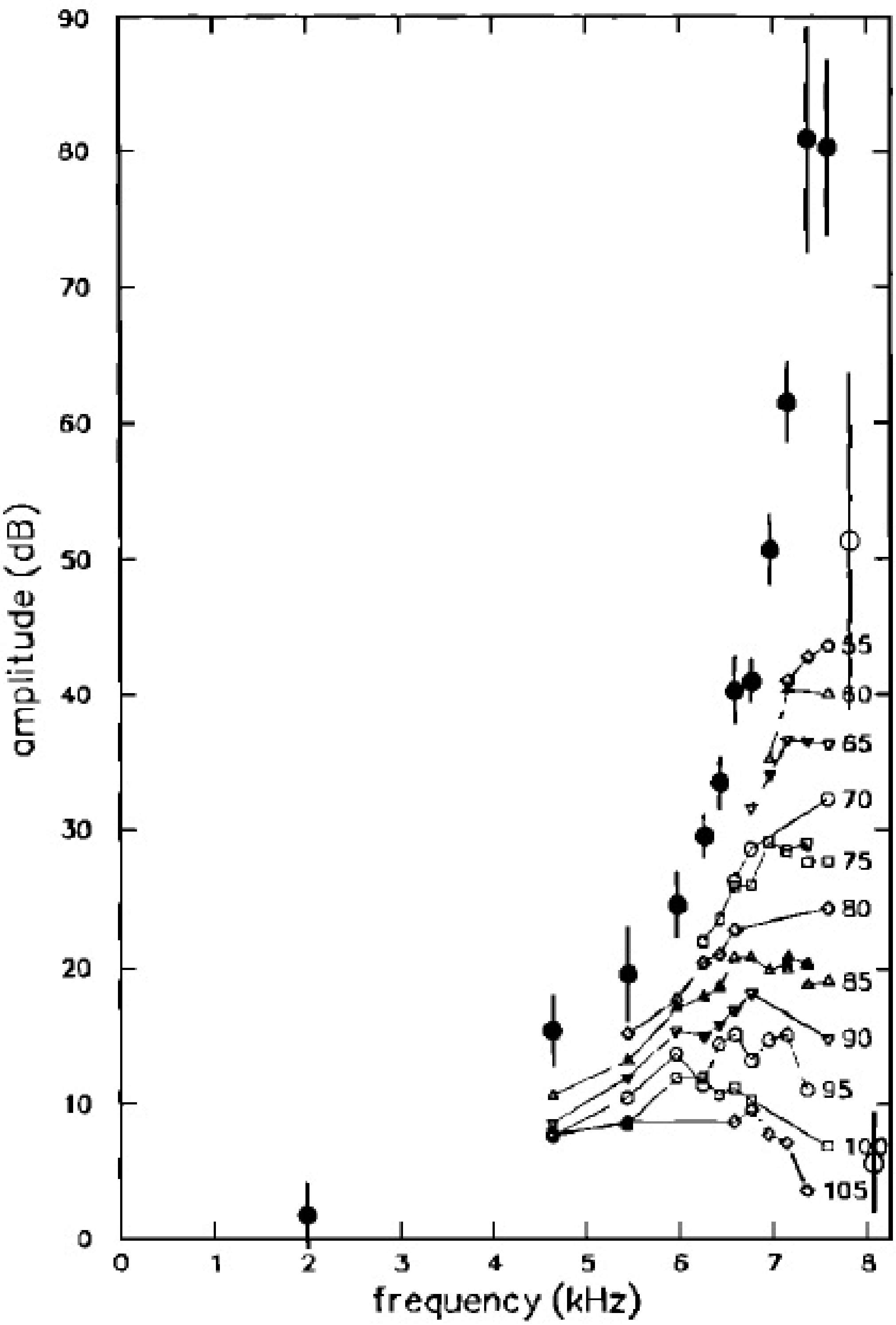
The Wave Equation:

$$P \propto \frac{1}{f\,Y}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\,\mathrm{e}^{ik\bar{x}}\,kl\tanh kh\;\tilde{P},$$
$$\tilde{P} \equiv \int_{-\infty}^{\infty} dk\,\mathrm{e}^{-ik\bar{x}}\,P,$$
$$P \propto T/Y,$$

where $T(f,x) \equiv \lim_{|V_\mathrm{m}|\to 0} D(f,x,V_\mathrm{m})$.

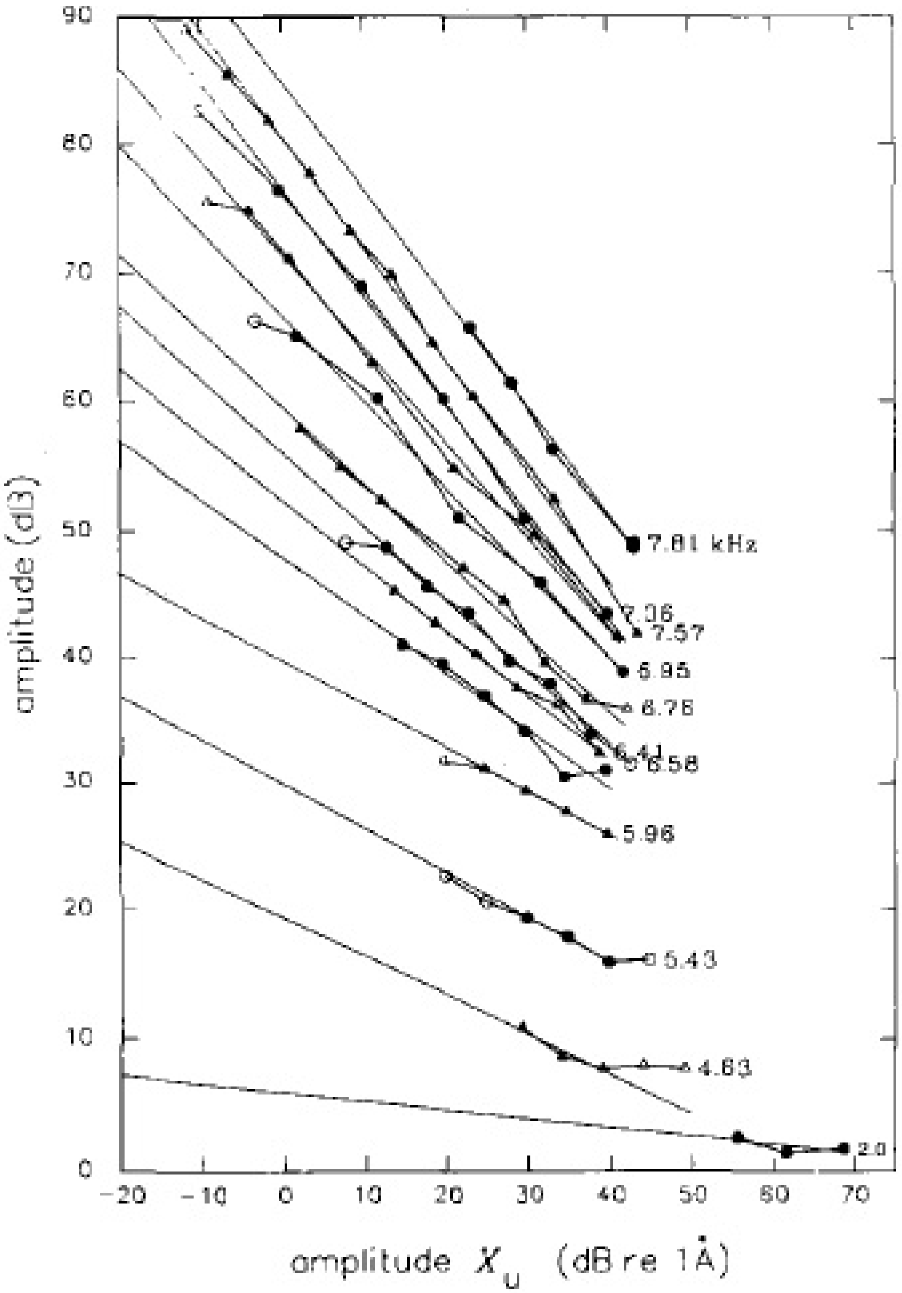
- Find T from the Ds.
- "Invert" T to find Y (the oscillator equation)
  - Long wavelengths everywhere ($|kh| \to 0$),
  - Short wavelengths for $x \approx \hat{x}$ $\quad(|kh| \to \infty)$.
- Now knowing Y, solve the full wave equation ($0 < |kh| < \infty$).

First find T from Ds:



Log $|D(f,V_{\mathrm{m}})|$ at fixed x for different sound pressure levels in dB, and the extrapolated transfer function $T(f)$ (data from W. Rhode, unpublished).

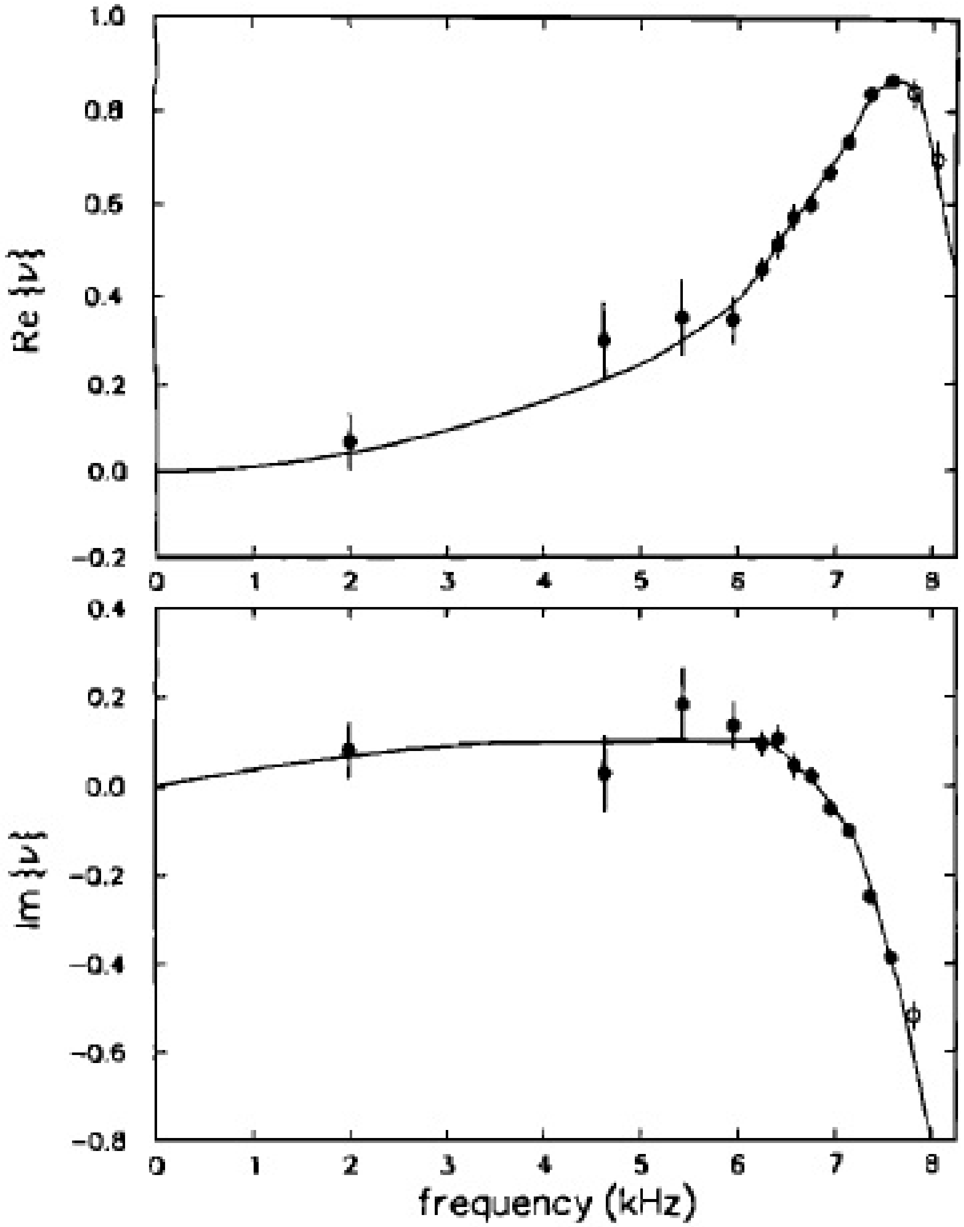
$$|D(f, V_\mathrm{m})| \propto |V_\mathrm{m}|^{-a(f)}$$



Amplitude of D at different frequencies as a function of the malleus amplitude $X_\mathrm{m} = V_\mathrm{m}/i\omega \equiv X_\mathrm{u}$. A horizontal line indicates a linear response.

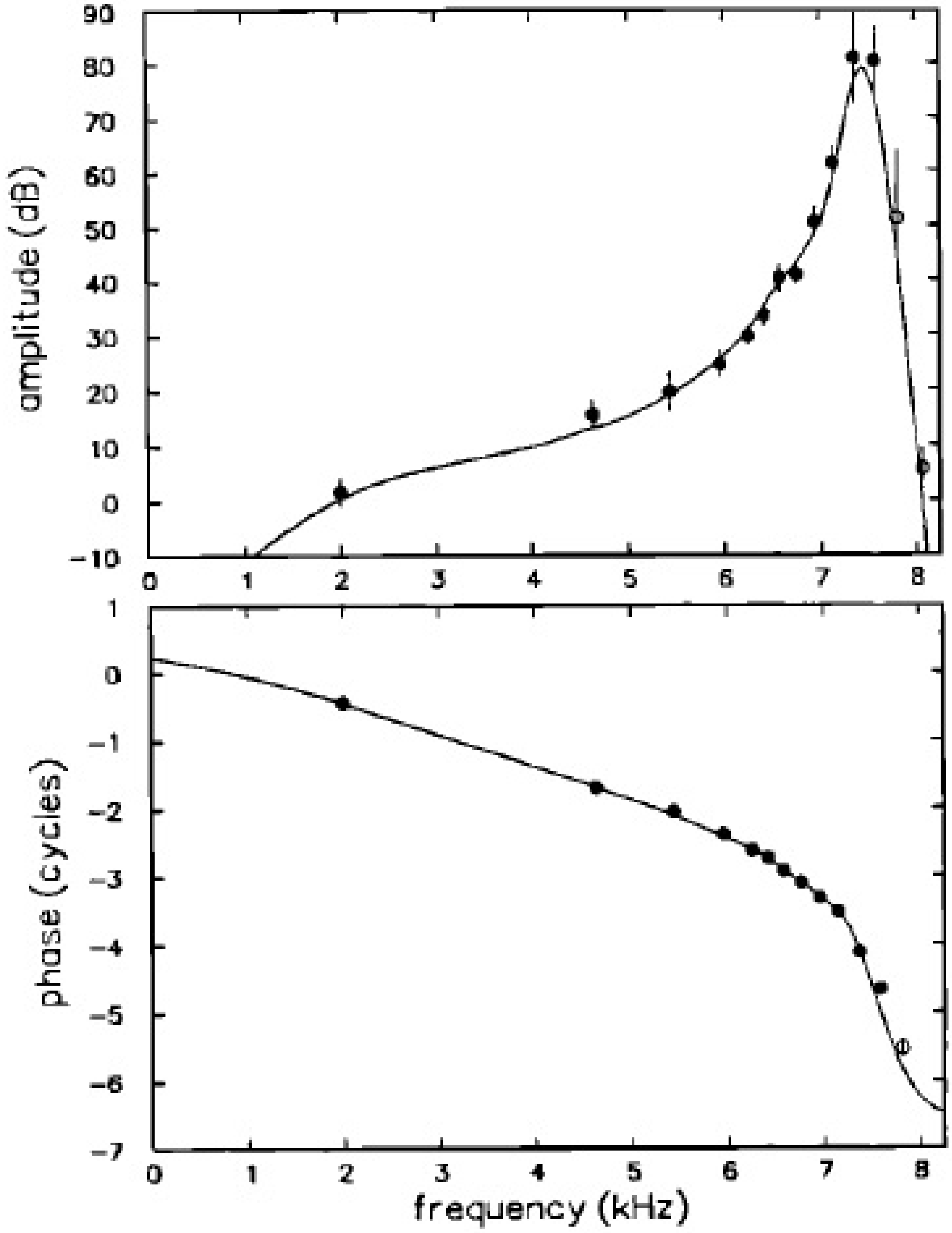
Combine two slopes into a single function:

$$\nu(f) \equiv a(f) + b(f)\, i.$$



Top: Slopes $a(f)$ of amplitude curves.
Bottom: Slopes $a(f)$ of phase curves.

$$a(f) = \text{Hilbert Transform}\,\{b(f)\}.$$



Transfer function $T(f)$ found by extrapolating the smoothed describing functions to the linear regime. The lines are a minimum-phase fit to $T(f)$. Points above the phase curve would violate causality.

$$T \equiv \mathrm{e}^{\alpha + i\theta},$$

$$\theta \leqslant \mathrm{HilbertTransform}\{\alpha\}\,.$$

Secret weapon!

"Invert" T to find Y:

First assume wavelengths are long ($|kh| << 1$):

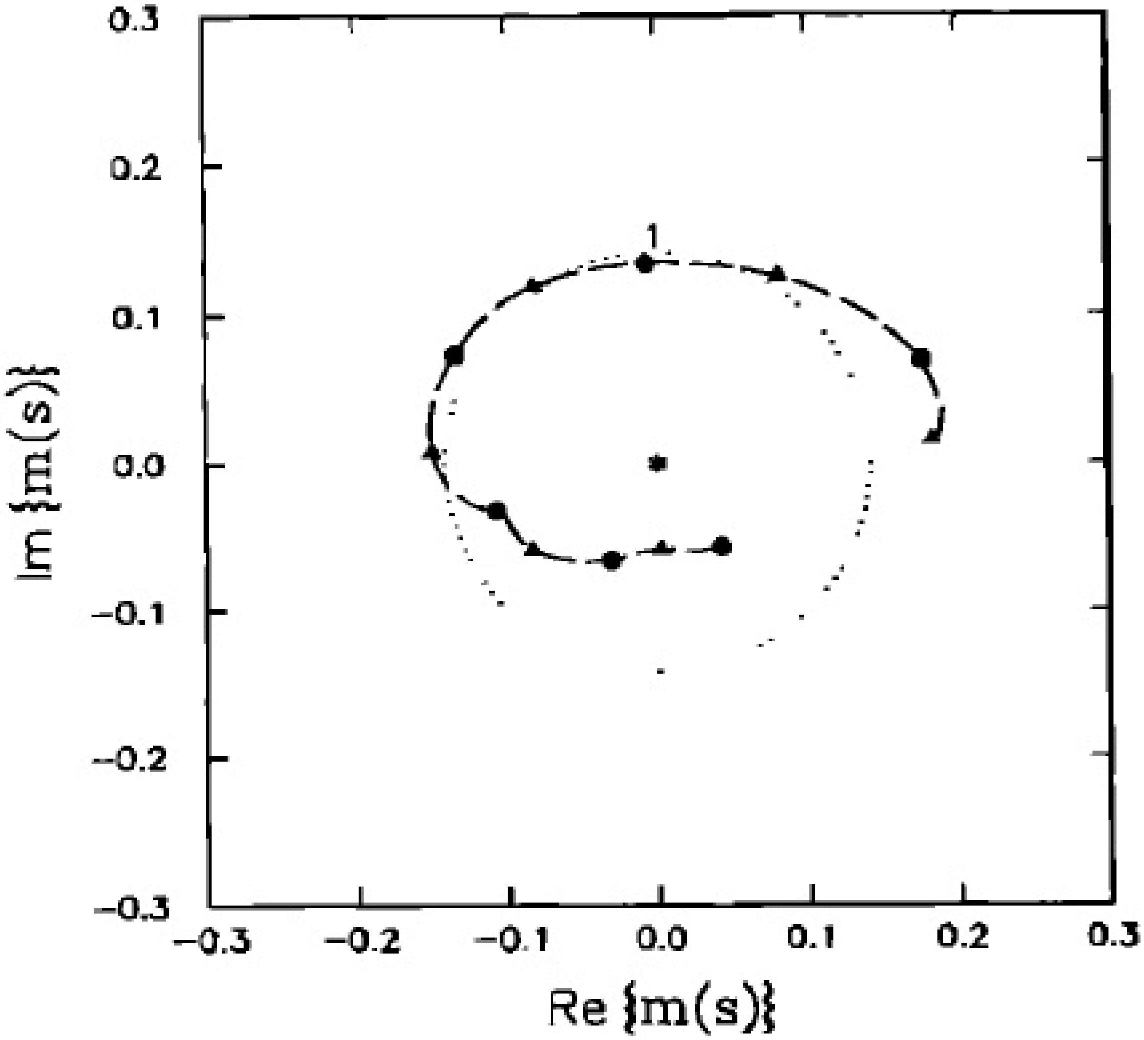
$$T\big|_{\text{measured}} \propto \exp\left[-2\pi i \int \frac{dx}{\lambda_{\text{long}}} + \cdots\right],$$
$$\frac{1}{\lambda_{\text{long}}} \propto N\sqrt{Y_{\text{long}}}.$$

Damped harmonic oscillator: Let $\beta \equiv f/f_c(x)$,

$$Y_{\text{harmonic osc.}} \propto \frac{\beta}{1 + i\delta\beta - \beta^2}.$$

Try:

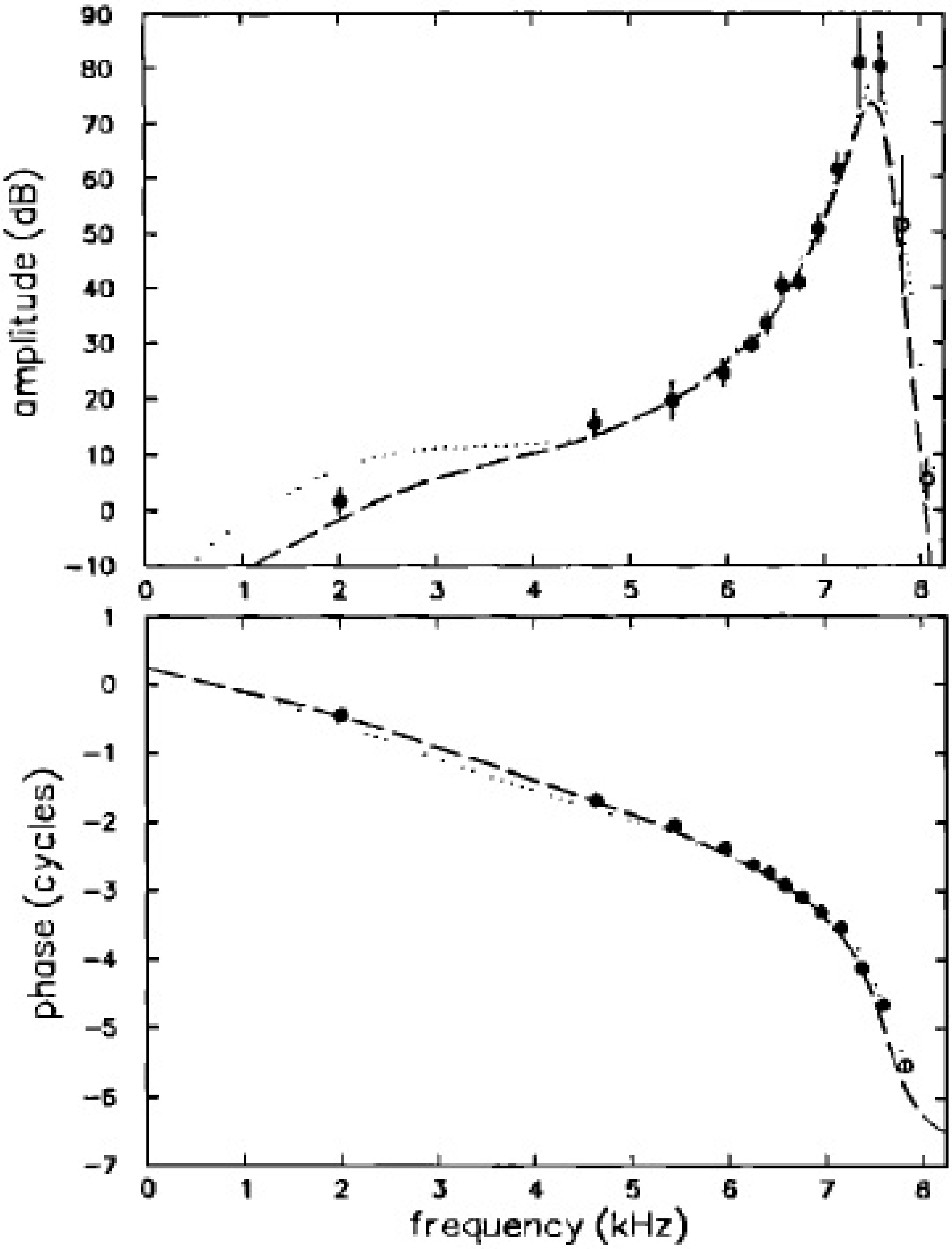
$$Y_{\text{long}} \propto \frac{\beta}{1 + i\delta\beta - \beta^2 + m(i\beta)}.$$



Dashed line: Parametric plot of missing piece $m(s \equiv i\beta)$ when $\delta \approx -0.12$. As β increases linearly, $m(i\beta)$ moves almost uniformly along a circle at a rate ψ.
Dotted line: $m(i\beta)$ simplified to $\rho \mathrm{e}^{-i\psi\beta}$.

If $\delta \approx -0.12$, then $m(i\beta) \approx \rho \mathrm{e}^{-i\,\psi\,\beta}$ where

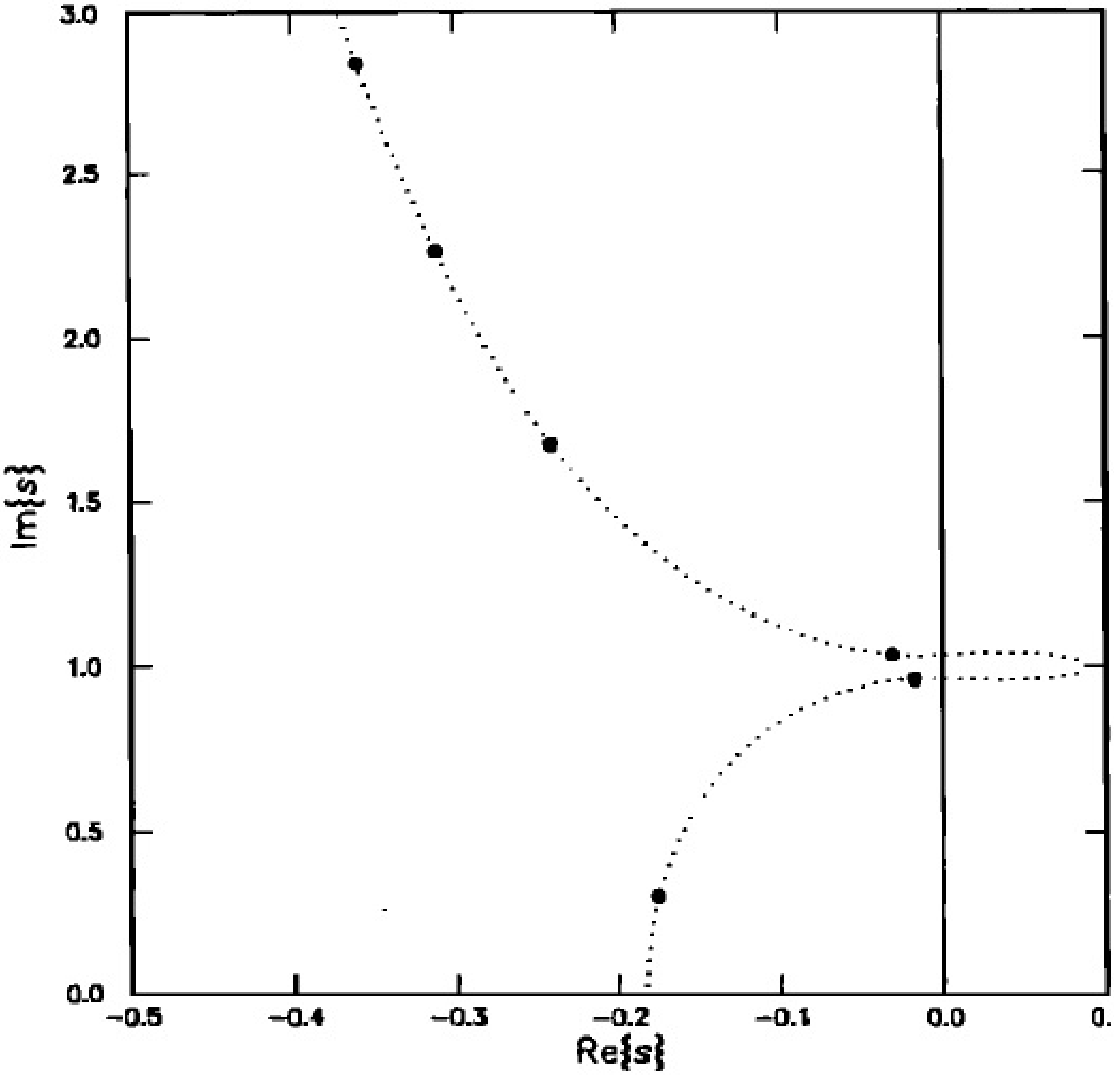
$$\rho \approx 0.12, \text{ and } \psi/2\pi \approx 1\frac{3}{4}.$$



Dashed line: T with the empirical admittance $Y_{\rm long}$. Dotted line: T when $Y_{\rm long}$ is simplified to $Y_{\rm long} \propto \beta/(1 + i\delta\beta - \beta^2 + \rho\mathrm{e}^{-i\psi\beta})$, $N \approx 5$.

Oscillator equation: Let $v_{\rm bm}$ = F.T. of $V_{\rm bm}$, and $\tau \equiv 2\pi f_c t$.

$$\frac{d^2 v_{\rm bm}}{d\tau^2} + \delta\,\frac{dv_{\rm bm}}{d\tau} + v_{\rm bm} \;=\; c_0\,\frac{dp}{d\tau} - \rho\, v_{\rm bm}(\tau - \psi).$$



Location of poles of $Y_{\rm long}$ in the $s \equiv i\beta$ plane.

$$\frac{1}{\lambda_{\rm long}} \propto N\sqrt{Y_{\rm long}}.$$

Make the two poles coalesce:

$$\frac{1}{\lambda_{\rm long}} = \frac{r(\hat{\beta})}{\beta - \hat{\beta}} + \cdots,\ |\beta - \hat{\beta}| << 1.$$

Now assume wavelengths are short ($|kh| >> 1$):

$$T|_{\text{measured}} \propto \exp\left[-2\pi i \int \frac{dx}{\lambda_{\text{short}}}\right],$$

where

$$Y_{\text{short}} \propto \frac{1}{\lambda_{\text{short}}}.$$

Since

$$\begin{aligned} T|_{\text{long}} &= T|_{\text{measured}}, \\ T|_{\text{short}} &= T|_{\text{measured}}, \end{aligned}$$

$T|_{\text{short}} = T|_{\text{long}}$ and $\lambda_{\text{short}} = \lambda_{\text{long}} + \cdots$.

Therefore,

$$Y_{\text{short}} \propto \frac{r(\hat{\beta})}{\beta - \hat{\beta}} + \cdots\ ,\ |\beta - \hat{\beta}| << 1.$$

$$\begin{aligned} \hat{\beta} &\equiv \hat{\beta}_r + \hat{\beta}_i\, i, \\ r &\equiv r_r + r_i\, i \end{aligned}$$

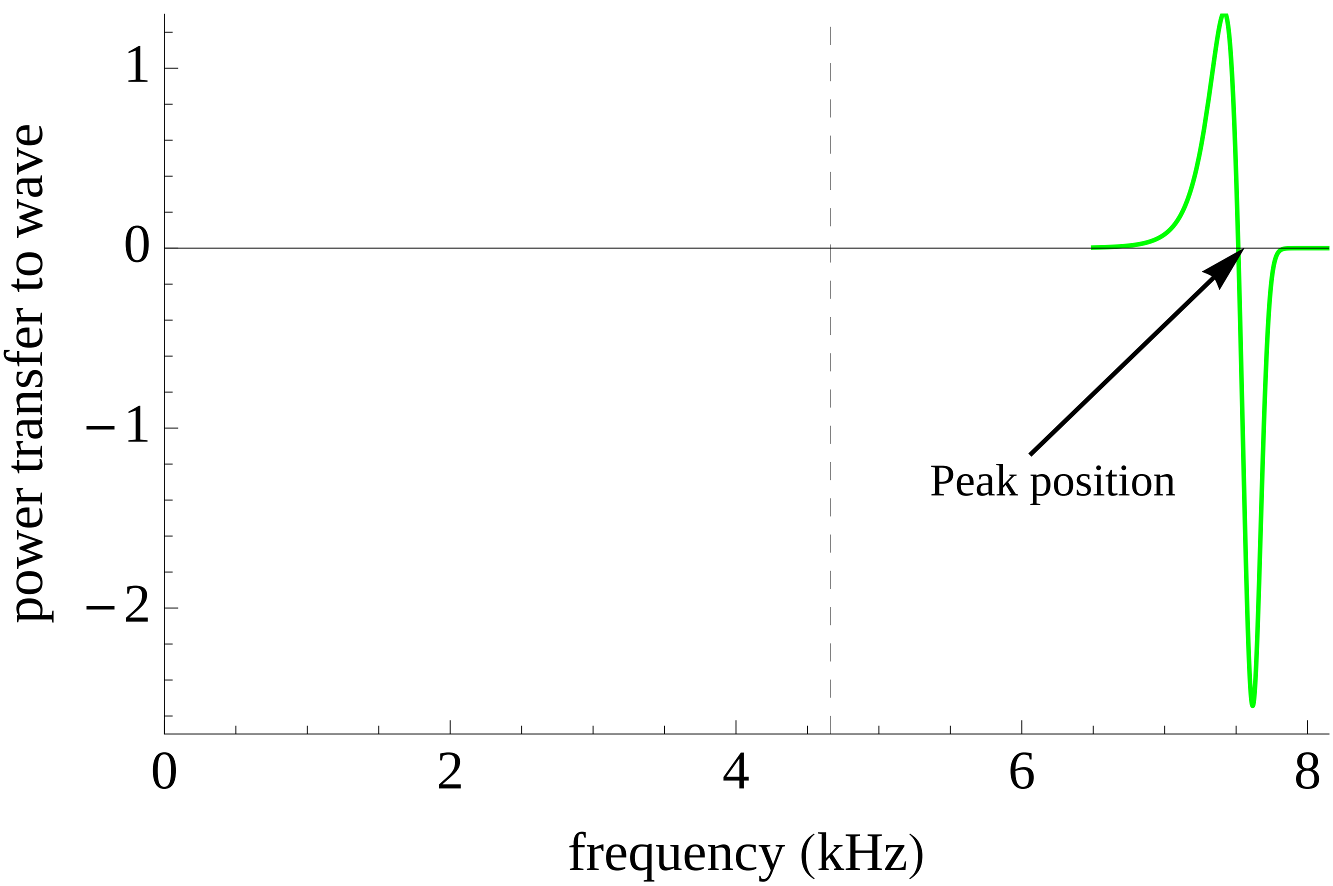
$$\frac{d^2 v_{\text{bm}}}{d\tau^2} + 2\hat{\beta}_i \frac{dv_{\text{bm}}}{d\tau} + \hat{\beta}_r^2 v_{\text{bm}} \propto r_r p + r_i \frac{dp}{d\tau},$$

$$Y \approx Y_{\text{short}} \quad \text{for } \beta \approx \hat{\beta}.$$

$$\begin{aligned} Y &\propto Y_{\text{short}} + c + \cdots \quad \text{for all } \beta \\ &= \frac{r(\hat{\beta})}{\beta - \hat{\beta}} + c + \cdots . \end{aligned}$$

c related to N.

$$< W >= \text{Real}\left\{\frac{1}{Y}\right\}\frac{|V_{\text{bm}}|^2}{2}.$$



Time-average power transfer per unit length to the traveling wave.

Solve the wave equation for $x \approx \hat{x}$:

$$Y = \frac{\alpha(\hat{\beta})}{x - \hat{x}(\hat{\beta})} + \cdots .$$

$$\text{Equation}: \;\; P \propto \frac{1}{f\, Y} \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, \mathrm{e}^{ik\bar{x}}\, kl \tanh kh \;\tilde{P}.$$

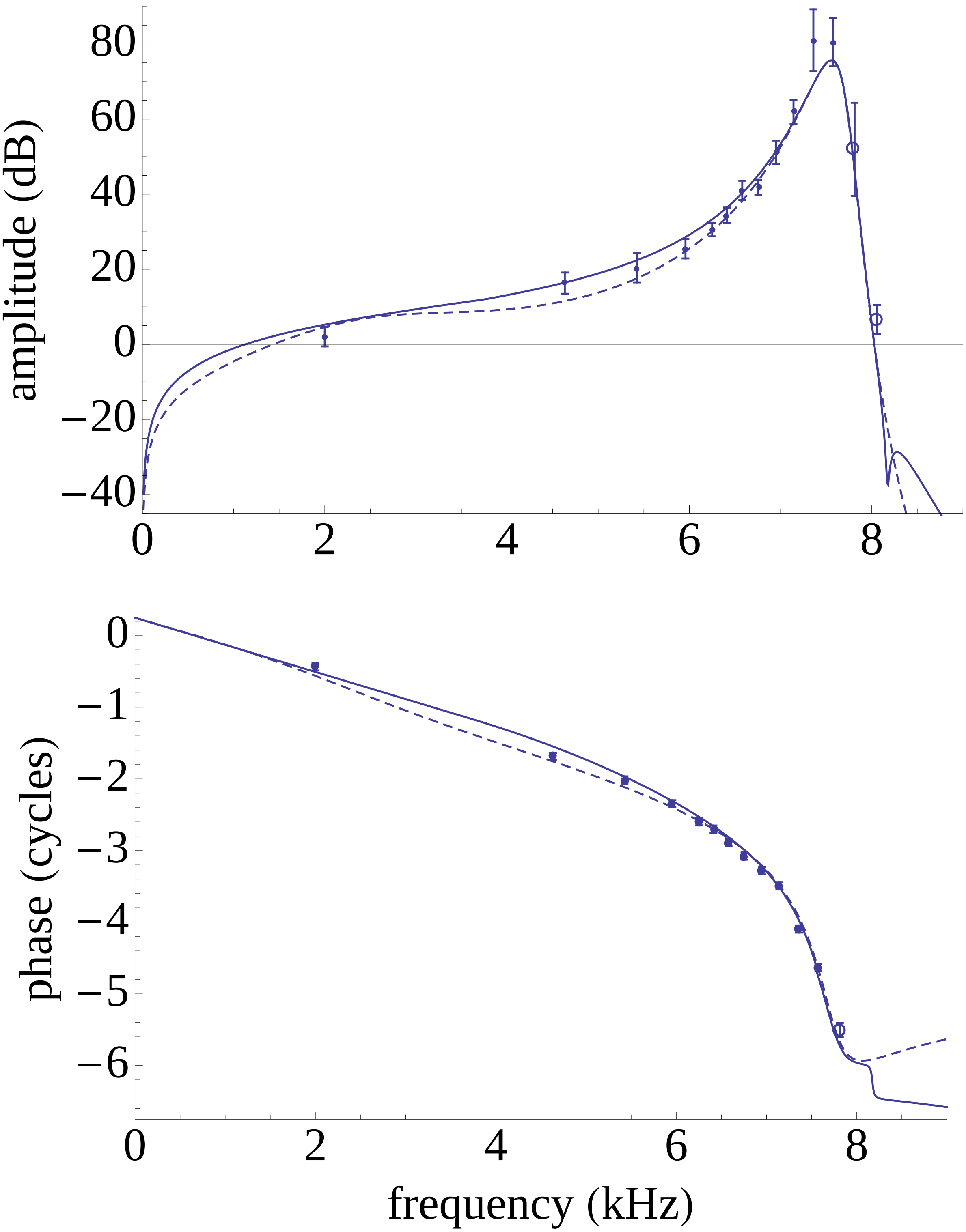
$$\text{Solution}: \;\; P \propto \frac{\bar{x}}{\epsilon} \mathcal{Z}\left(\frac{\bar{x}}{\epsilon}, \alpha\right),$$

where $\mathcal{Z}$ is a new special function:

$$\mathcal{Z}(x,\alpha) \equiv \int_0^{\infty} \frac{dk}{2\pi} \exp\left[-ikx - i\alpha \int_k^{\infty} \frac{dk'}{k' \tanh\, k'h}\right],$$

$$\begin{aligned}
\hat{\beta} \;&=\; 0.98 + 0.028\, i, \\
c \;&=\; 38.5, \\
\epsilon(\hat{\beta}, c) \;&=\; h/l, \\
&=\; 0.11 \;\; (\textit{prediction}), \\
\alpha(\hat{\beta}) \;&=\; -4.29 - 4.79\, i.
\end{aligned}$$

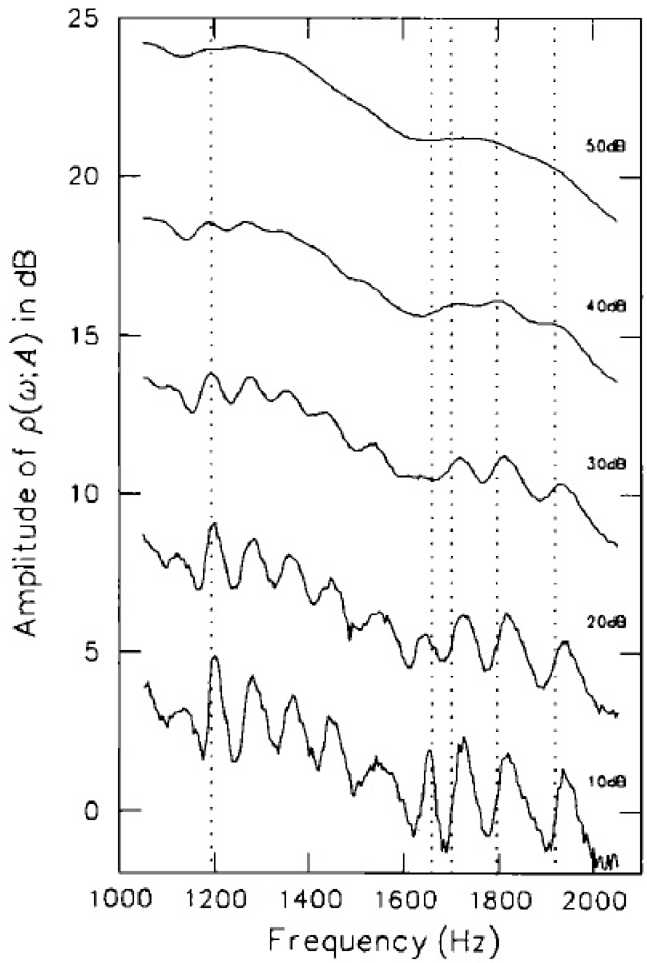
The only inputs are $\hat{\beta}$ and c!



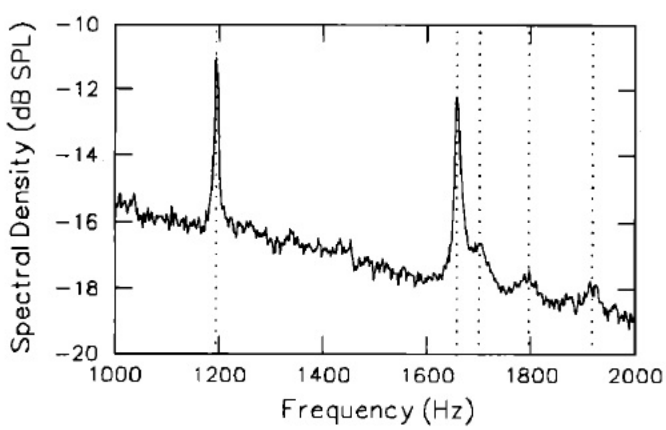
The complete chimeric transfer function including the effects of long, short, and intermediate wavelengths. The dashed line is the one-dimensional time-delay transfer function.

T for $f > 7.5$ kHz predicted.

Measure and compute sensations:



Pressure in human ear canal with tones.

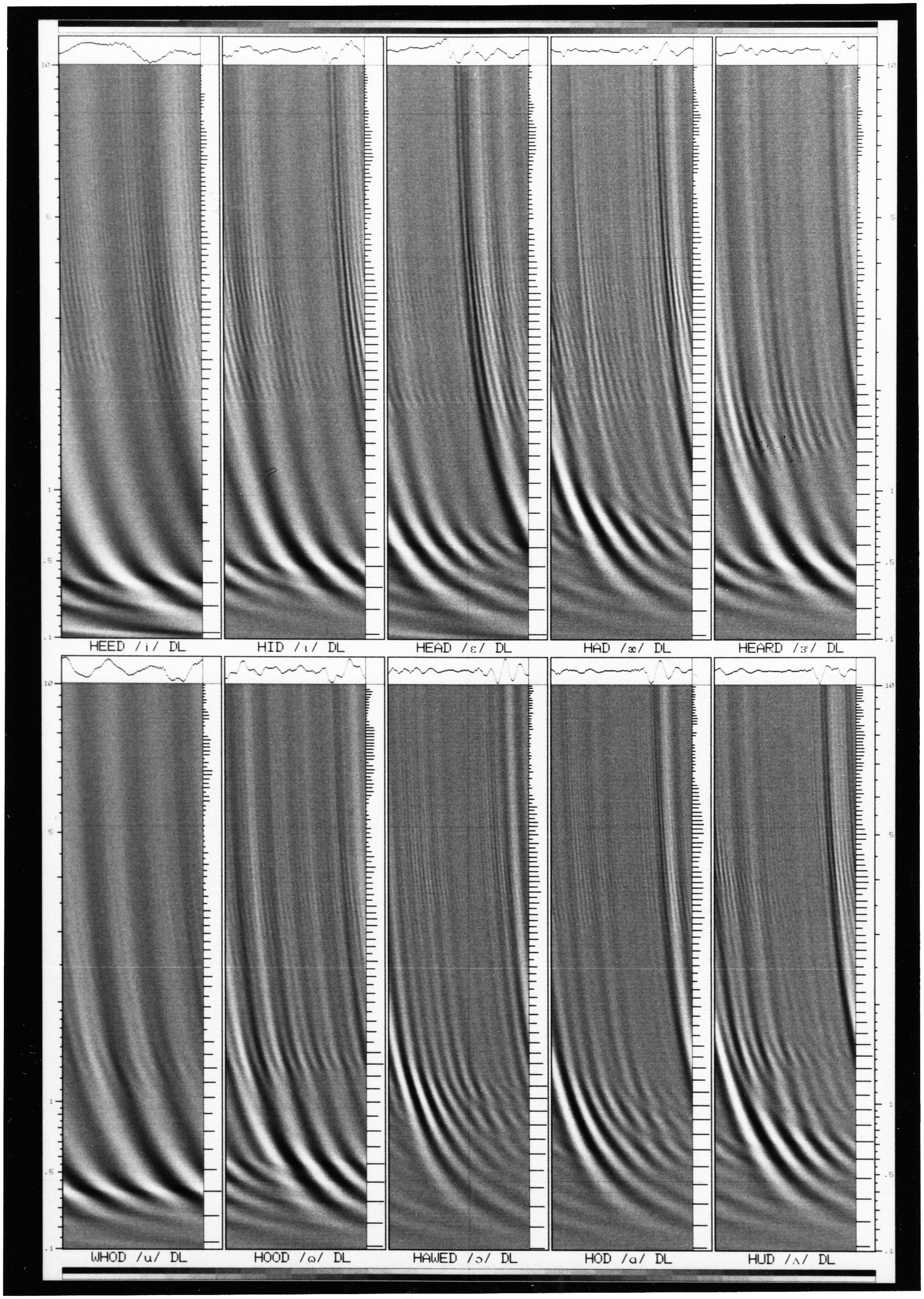


Energy density in human ear canal without any stimulus (with C. Shera).

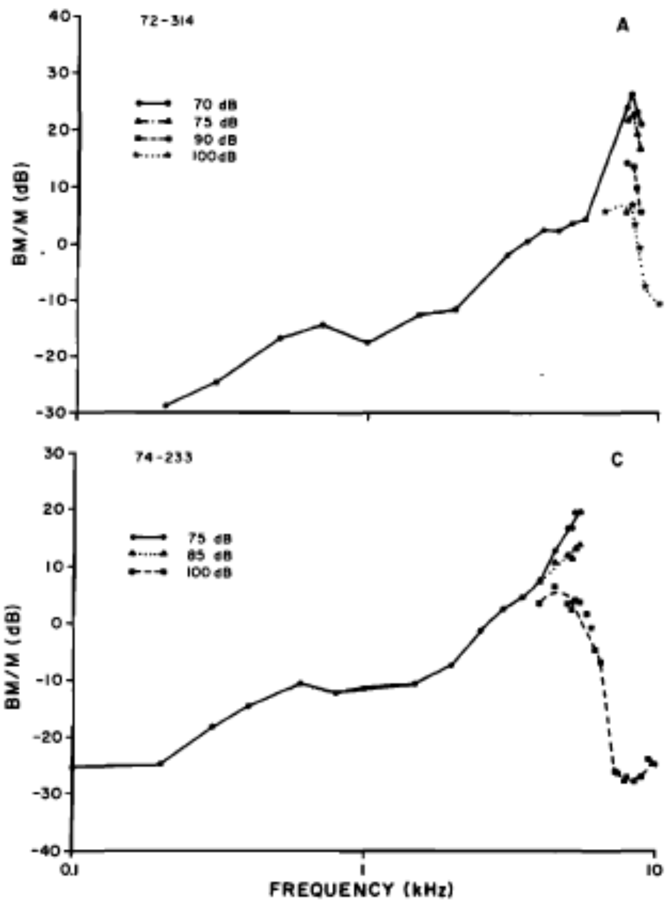
Applications:

1. Tests for hearing loss
2. Hearing aids
3. Cochlear implants
4. Speech & music compression algorithms
5. Music synthesizers
6. Signal processors (cochlear transforms)
7. Preprocessors for computer speech recognition

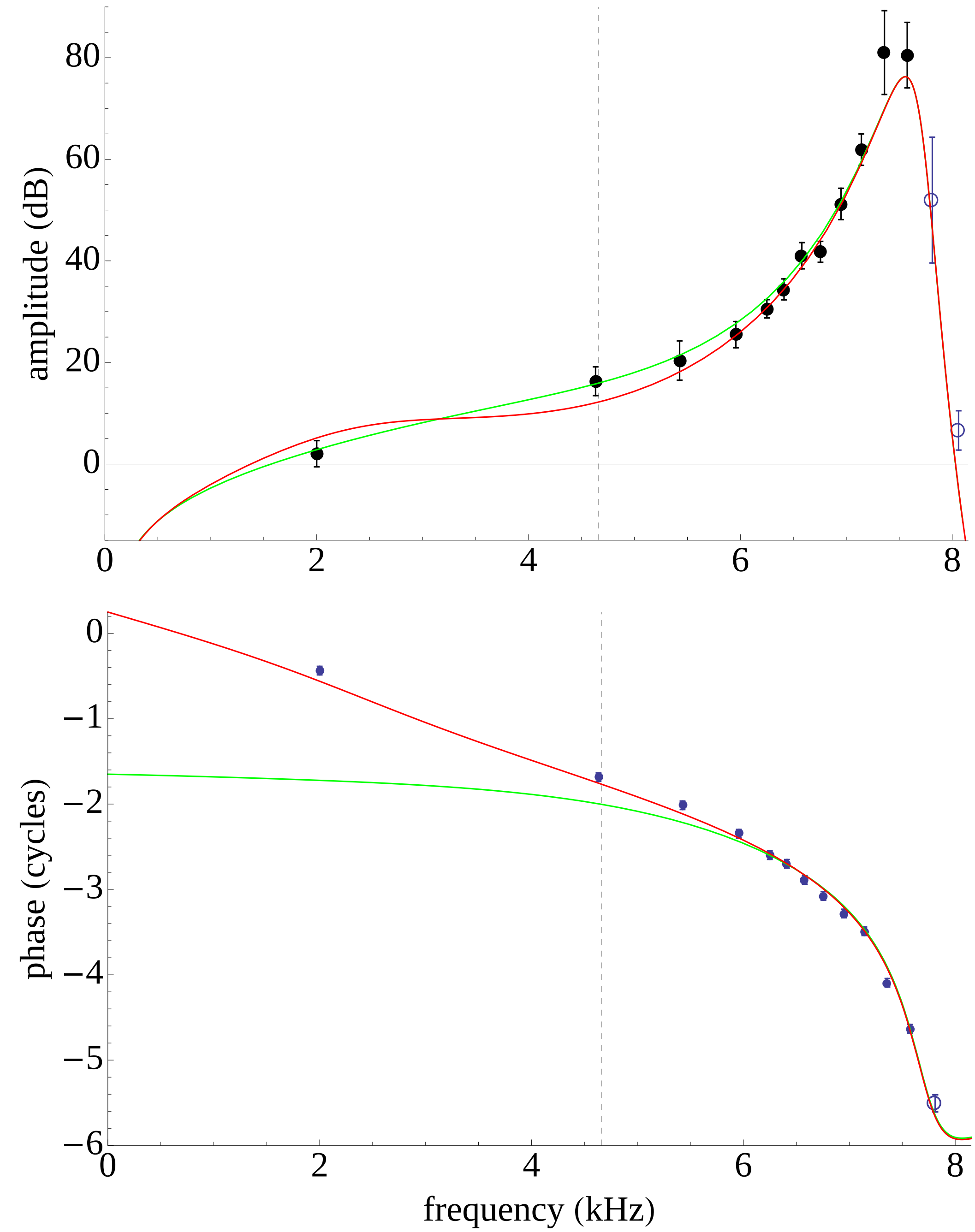
What's the connection with quarks, and what's the take-home message?



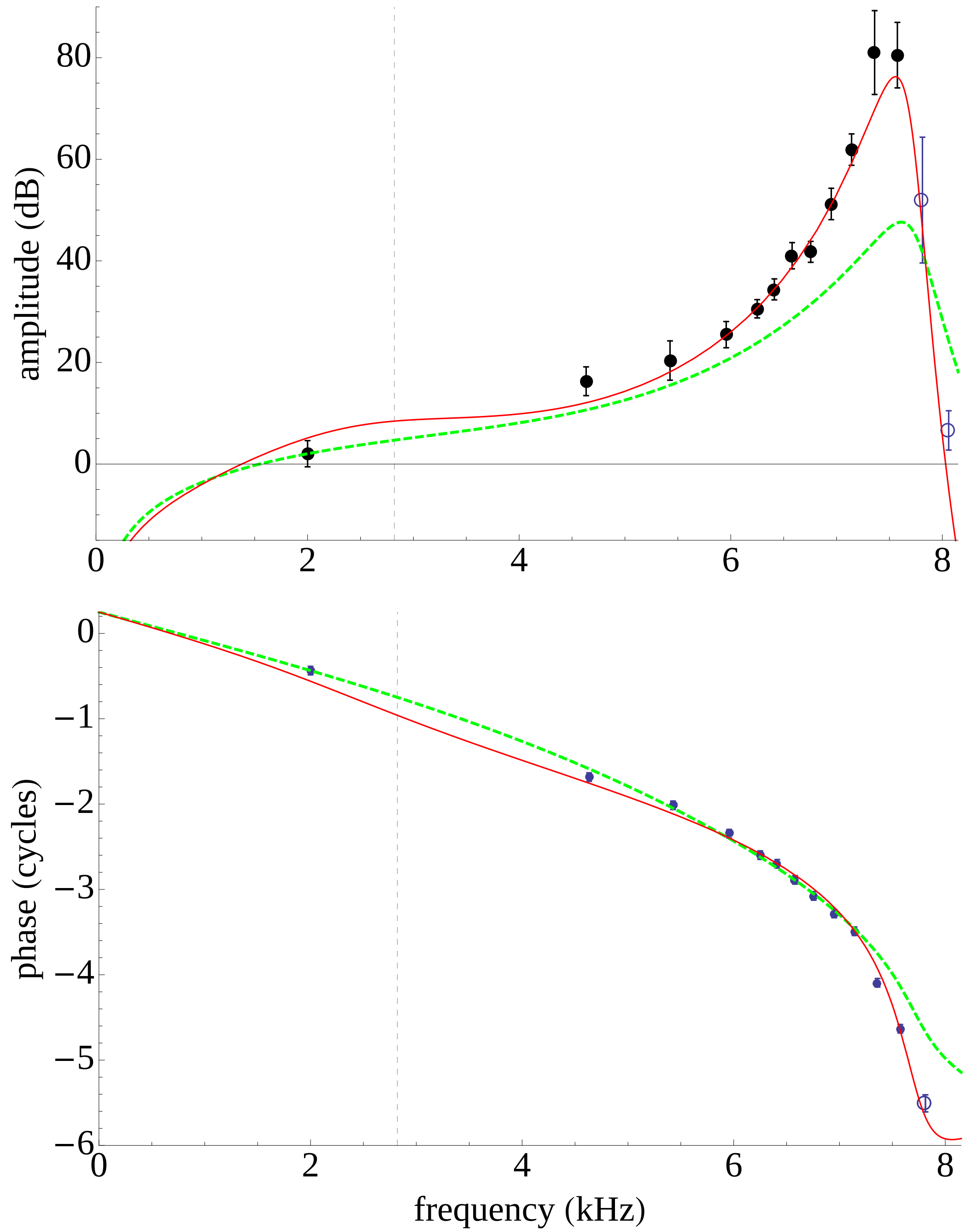
Waves for vowels (unpublished, 1976).



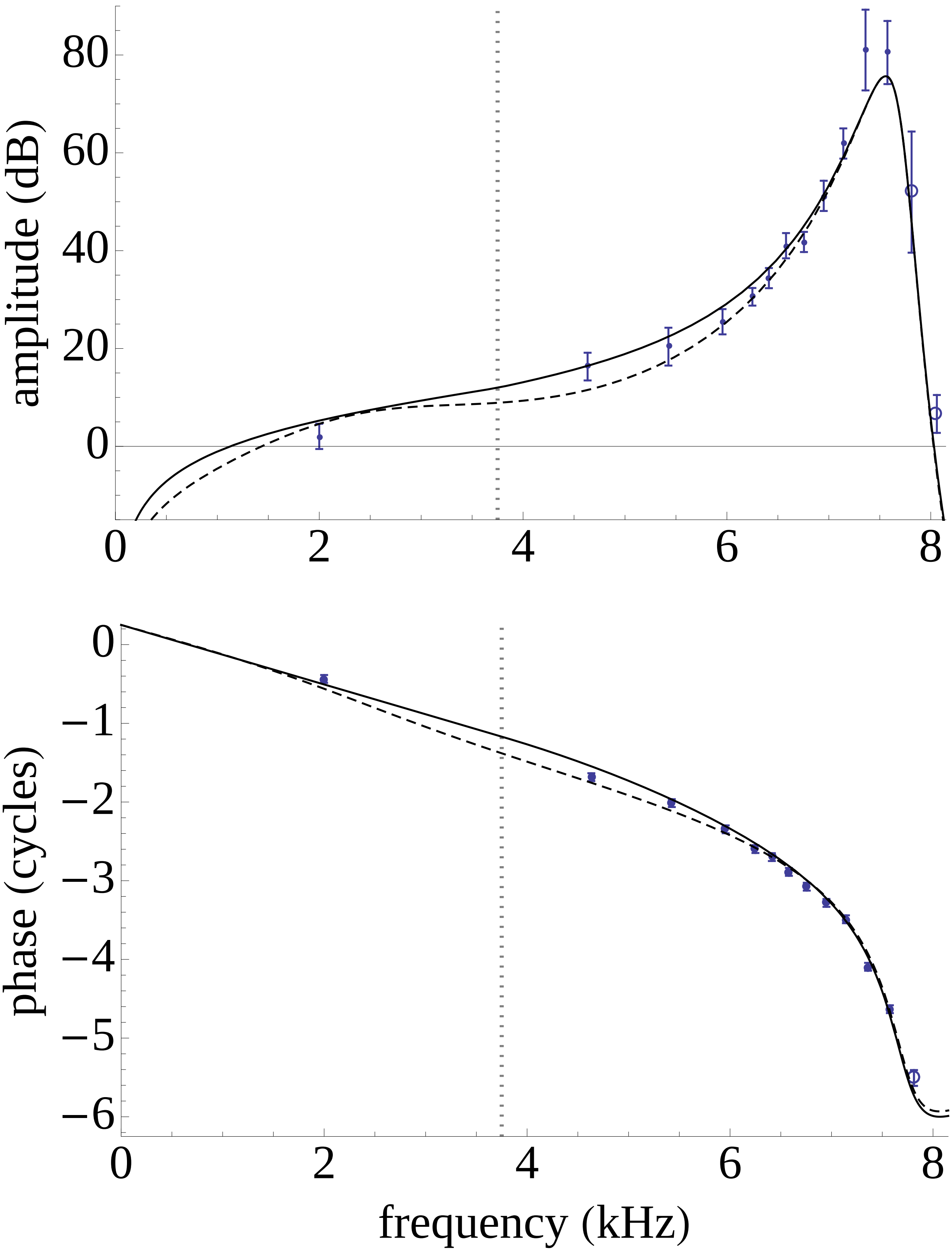
Log $|D(f, V_{\mathrm{m}})|$ at fixed x at different sound pressure levels in dB (data from W. Rhode, published).



Short-wavelength transfer function (green) derived from a long-wavelength transfer function fit (red).



Long-wavelength transfer function (dashed green lines) derived from the long-wavelength transfer function fit (red lines).



Transfer function (solid line): Long-λ TF for frequencies less than the transition frequency (vertical dotted line), and short-λ TF for greater frequencies. Time-delay TF (dashed line). $\epsilon \equiv h/l = 0.11$.